(Pages : 3)

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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, November 2012
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV
Complex Analysis and Linear Algebra (T A)

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions from Part **A** and **one full** question from **each** Module of Part **B**.*

PART – A 40

1. Show that the function $f(z) = xy + iy$ is continuous everywhere but is not analytic anywhere.
2. Prove that the function $\log(x^2 + y^2) + x - 2y$ is harmonic.
3. If $f(z)$ and $\overline{f(z)}$ are analytic, show that $f(z)$ is a constant.
4. Find the image of the strip $\frac{1}{2} \le y \le 1$ under $w = z^2$.
5. Evaluate $\int_0^{1+i} (x - y + ix^2)$ along the line from $z = 0$ to $z = 1+i$.
6. Expand $\frac{1}{z}$ about $z = 2$ as Taylor's series.
7. Evaluate $\int_C \frac{\tan z/2}{(z-a)^2} dz$, $-2 < a < 2$ where C is the boundary of the square whose sides are $x = \pm 2$ and $y = \pm 2$.
8. Let $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and $u = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$.

   Determine if u belongs the mill space of A.

P.T.O.

9. Find the coordinate vector of $X = (8, -9, 6)$ relative to the basis $\beta = \{(1, -1, -3), (-3, 4, 9), (2, -2, 4)\}$.

10. Find a least square solution of $AX = B$ where $A = \begin{bmatrix} 2 & 3 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

PART – B **(20×3=60 Marks)**

Module – I

11. a) Show that the function $f(z) = \frac{x^2 y^5 (x + iy)}{x^4 + y^{10}}; z \neq 0$ and $f(0) = 0$ satisfies CR equations at the origin but $f'(0)$ does not exist.

b) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.

c) Find the image of $|z - 3i| = 3$ under $w = \frac{1}{z}$.

12. a) Find the analytic function $f(z) = u + iv$ if $u - v = e^x(\text{Cosy} - \text{Siny})$.

b) If $f(z) = u + iv$ is analytic, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|u|^2 = 2|f'(z)|^2$.

c) Determine the bilinear transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ into $w_1 = -3, w_2 = -1, w_3 = 1$ respectively.

Module – II

13. a) Verify Cauchy's Theorem for $\int_C z^2 dz$ where C is the boundary of the triangle with vertices (0, 0), (2, 0), (0, 2).

b) Evaluate f(2) and f(3) where $f(a) = \int_C \frac{2z^2 - z - 2}{z - a} dz$ and C is $|z| = 2.5$.

c) Expand $f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)}$ in a Laurent's series if $|z| > 3$.

14. a) Find the poles and residues of $\frac{\sinh z}{z^4}$.

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$ by contour integration in the complex plane.

c) Show that $\int_0^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$.

Module – III

15. a) Find the LU decomposition of $A = \begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix}$.

Hence solve AX = B when $B = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$.

b) Find the projection of u = (1, 1, 2) along v = (1, 2, –4) in R^3.

c) Find an orthonormal basis for the subspace spanned by (1, 2, 1), (1, 0, 1) (3, 1, 0) in $\mathbb{R}^3$.

16. a) Find maxima or minima of $f(x_1, x_2) = x_1^3 - x_2^3 + 12x_1x_2$.

b) Determine the nature of quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$.

c) Obtain the singular value decomposition of $A = \begin{bmatrix} 7 & 1 \\ 0 & 0 \\ 5 & 5 \end{bmatrix}$.
